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NORM2L

AN INTERACTIVE COMPUTER PROGRAM
FOR ACOUSTIC NORMAL MODE CALCULATIONS
FOR THE PEKERIS MODEL

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NORM2L

AN INTERACTIVE COMPUTER PROGRAM FOR ACOUSTIC NORMAL MODE CALCULATIONS FOR THE PEKERIS MODEL

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December 1980

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ABSTRACT

The interactive computer program, NORM2L, calculates the discrete normal modes and acoustic propagation loss for the Pekeris model of the ocean. The Pekeris model is a simple two-layer model in which the two layers represent the seawater and seabed. For many shallow-water environments, the model is a reasonable approximation to the actual physical situation and can be used to investigate acoustic propagation at low frequencies.

For ease of future expansion and modification, the program NORM2L is written in modular form in FORTRAN. The results of NORM2L are compared with those of other computer programs.

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RESUME

Le programme d'ordinateur interactif NORM2L sert à calculer les modes normaux discrets et la perte de propagation acoustique pour le modèle de Pekeris de l'océan. Le modèle de Pekeris est un modèle simple à deux couches dans lequel les deux couches représentent l'eau de mer et le fond de la mer. Pour plusieurs environnements en eau peu profonde, ce modèle est une approximation raisonnable de la situation réelle et peut servir à étudier la propagation des ondes acoustiques aux basses fréquences.

Pour en faciliter l'expansion et la modification futures, le programme NORM2L est écrit sous forme modulaire en langage FORTRAN. L'auteur compare les résultats du NORM2L à ceux des autres programmes d'ordinateur.

TABLE OF CONTENTS

1   INTRODUCTION

NORM2L is an interactive computer program which calculates the discrete normal modes and propagation loss for the Pekeris [1] acoustic model described in section 2.1. Although this model is simple, it contains all the essential features of normal mode theory, and for many shallow water environments is a reasonable approximation to the actual physical situation.

The main reason for writing NORM2L was to have a convenient program for becoming acquainted with normal mode calculations. The interactive program and simple model allow rough answers to be obtained quickly and cheaply. Since many of the solutions are known analytically, the extensive calculations of a general normal mode program are not necessary. Also, the interactive nature of the program allows the user to see the results immediately. The user can then change the input parameters and look at the new set of results. In this way, the user can quickly get a feeling for the importance of the physical parameters.

Another reason for writing the program was to test new ideas and algorithms in normal mode theory in an easy to modify package. For example, to help verify that a numerical procedure is working or that a particular normal mode program has been implemented properly, the results from the analytic solutions of the Pekeris model can be compared with similar results from other normal mode programs which

obtain their results numerically. In one application a slightly modified version of NORM2L was found valuable in testing and debugging a normal mode program based on a two-ended shooting method [2,3]. In another application, not foreseen at the time NORM2L was written, the program was modified and some of the output used for mode enhancement in a preliminary analysis of some real data [4]. The present report does not discuss these applications, but restricts itself to a description of the program.

The main body of the report contains a general description of the model and program, with details relegated to the appendices. Section 2 contains the theoretical basis with a summary of the Pekeris model and a description of the algorithm used to calculate the normal mode wave numbers. Section 3 contains a general description of the program, its structure and important variables, and results of testing and verification. A number of tables are presented on wave numbers, mode functions, and propagation loss to be used as reference for checking the accuracy of new or other normal mode programs. In Section 4, the recommendations contain some possible future extensions and uses of the program.

Appendices A to D contain further details on the program. Appendix A is a user's guide for NORM2L with details on the hardware-software environment, input and output, operational instructions, restrictions and limitations, functions and subroutines, accuracy, memory

requirements, and execution times. Appendices B and C

present sample input/output from the terminal and line

printer, and Appendix D contains a program listing.

Appendices E and F are forward-looking, giving some

anticipated modifications and extensions and a place for the

user to add his comments and suggestions. Some of these

changes will be incorporated in future versions of NORM2L.

2.   THEORETICAL BASIS

2.1   The Pekeris Model

Figure 1 illustrates the environment described by the Pekeris [1] acoustic normal mode model. The model has a homogeneous layer of thickness h bounded above by a pressure release surface and bounded below by a homogeneous semi-infinite half-space of greater sound speed. The sound speeds c_1 and c_2 and the densities ρ_1 and ρ_2 are constants. To make the environment more realistic, the bottom layer has been given an absorption coefficient a_2. A source at depth z_s in the first layer is operating at frequency f. The problem is to calculate the propagation loss between the source and a receiver at depth z_r and horizontal distance r from the source.

The greater sound speed of the bottom causes some of the acoustic energy to be totally reflected from the bottom. The successive reflections from the surface and bottom cause the water layer to act as a waveguide, and a standing wave pattern is set up in the depth direction. These standing waves are the normal modes. Near the source some of the energy strikes the bottom at angles less than the critical angle and is partially transmitted. This energy corresponds to the continuous spectrum of normal mode theory. Since this energy leaks out to the bottom after several bottom reflections, it is nearly always neglected in normal mode calculations. Like most normal mode programs, NORM2L

calculates only the discrete normal modes. This means that the model is not applicable when the source and receiver are separated by horizontal distances less than several water depths.

A standard reference for underwater acoustics is Tolstoy and Clay [5], although the formulae for the normal mode equations are scattered throughout the book. Chapter 9 of the newer book by Clay and Medwin [6] contains a fairly clear introduction to normal mode theory. Only the essential equations are presented here. Note that the formulation is in terms of the velocity potential, and that only the discrete modes are calculated.

For a frequency f (angular frequency $\omega = 2\pi f$) the normal modes are the eigenfunctions $u_n(z)$ of the following differential equation

$$\frac{d^2 u_n}{dz^2} + [\frac{\omega^2}{c^2} - k_n^2] u_n(z) = 0 \ . \tag{1}$$

Once the eigenvalue k_n is determined (see below), the solutions u_n for the two layer model are given as follows:

$$u_n(z) = A_n \sin(\gamma_{1n} z) \ , \qquad\qquad 0 \leq z \leq h \tag{2a}$$

$$u_n(z) = (\rho_1 / \rho_2) A_n \sin(\gamma_{1n} h) \exp[-\gamma_{2n}(z-h)] \ , \qquad z > h \tag{2b}$$

where

$$\gamma_{1n} = [\omega^2 / c_1^2 - k_n^2]^{1/2} \tag{3a}$$

$$\gamma_{2n} = [k_n^2 - \omega^2/c_2^2]^{1/2} \tag{3b}$$

and

$$A_n^{-2} = \frac{1}{2} \rho_1 h [1 - \frac{\sin(2\gamma_{1n}h)}{2\gamma_{1n}h} + \frac{\rho_1}{\rho_2} \frac{\sin^2(\gamma_{1n}h)}{\gamma_{2n}h}] \tag{4}$$

The normalization factor A_n is determined from:

$$\int_0^\infty \rho(z)u_n(z)u_m(z)dz = \delta_{nm} \tag{5}$$

where δ_{nm} is the Kronecker delta function.

The conditions of continuity of pressure and particle velocity require that at the interface z=h:

$$\rho_1 u_n(h^-) = \rho_2 u_n(h^+) \tag{6a}$$

$$\frac{du_n(h^-)}{dz} = \frac{du_n(h^+)}{dz} \tag{6b}$$

Condition 6a has already been incorporated into the equation for u_n. Condition 6b gives the equation used to determine k_n,

$$\tan \{h[\omega^2/c_1^2 - k_n^2]^{1/2}\} = - \frac{\rho_2}{\rho_1}[\frac{\omega^2/c_1^2 - k_n^2}{k_n^2 - \omega^2/c_2^2}]^{1/2} \tag{7}$$

Solution of this equation leads to a set of discrete values for k_n:

$$\omega/c_1 > k_1 > k_2 > \ldots > k_N > \omega/c_2 \tag{8}$$

If there is attenuation in the bottom, the wave number of the sound speed in the bottom can be written as

the complex number $k_B = \omega/c_2 + ie$. Usually e is quite small
in comparison to ω/c_2 and perturbation theory can be used
[7]. Then,

$$k_n \rightarrow k_n + i \, \delta_n \tag{9}$$

where,

$$\delta_n = e \, \frac{\omega \rho_2}{c_2 k_n} \int_h^\infty |u_n(z)|^2 \, dz \tag{10}$$

For the two layer model the integral can be evaluated,
giving

$$\delta_n = e \, \frac{\omega \rho_1^2 A_n^2}{2\rho_2 c_2 k_n \gamma_{2n}} \tag{11}$$

Note that the absorption coefficient a_2 is usually given as
dB per unit length referenced to 1 kHz. Since the
attenuation is assumed to be linearly dependent on the
frequency, then e is given by

$$e = a_2 \, (f/1000) \, (\ell n \; 10)/20 \tag{12}$$

and has units of inverse length, the same as the wave number.
The units are usually called nepers per unit length.

There are a number of conventions for what is meant
by a unit source. The one used here will define a source of
strength Q as one in which the pressure $p(\underline{x})$ near the source
at $\underline{x}_0$ is given by

$$p(\underline{x}) = Q \, |\underline{x} - \underline{x}_0|^{-1} \, e^{ik_0|\underline{x} - \underline{x}_0|} \tag{13}$$

where $k_0 = \omega/c(\underline{x}_0)$. The pressure due to the discrete modes

is then given by:

$$p(\underline{x}) = i\pi Q\rho(z) \sum_{n=1}^{N} u_n(z_0)u_n(z)H_0^{(1)}(k_n r) \qquad (14)$$

and the propagation loss relative to unit distance,

defined by

$$PL = -10 \log_{10} |p(\underline{x})/p(\underline{x}_1)|^2 \qquad (15)$$

where $|\underline{x}_1 - \underline{x}_0| = 1$, is given by

$$PL = -10 \log_{10} |\pi\rho(z) \sum_{n=1}^{N} u_n(z_0)u_n(z)H_0^{(1)}(k_n r)|^2 \qquad (16)$$

This formula is valid for all depths z [8], although most

authors say it is valid only in layer one.

For $k_n r$ large (and no attenuation in the bottom),

the expression for the intensity $|p|^2$ can be written as,

$$|p(x)|^2 = |C|^2 \sum_{n=1}^{N} |u_n(z_0)u_n(z)|^2 (2/\pi k_n r)$$

$$+ |C|^2 \sum_{n \neq m}^{N} u_n(z_0)u_n(z)u_m(z_0)u_m(z) \frac{2}{\pi r(k_n k_m)^{1/2}} e^{i(k_n - k_m)r} \qquad (17)$$

where $C = i\pi Q\rho(z)$, and the Hankel function has been replaced

by its asymptotic form

$$H_0(k_n r) \simeq (2/\pi k_n r)^{1/2} e^{i(k_n r - \pi/4)} \qquad (18)$$

Notice that only the second term of (17) contains the

oscillating trigonometric terms. There are several reasons

for expecting this term to be small. First, if the r term

in the denominator is removed, the average value of the term

(as a function of range) is zero. Moreover, as a function
of z, the mode functions should appear more or less with
random amplitudes and signs. Thus the main features of
propagation loss are contained in the first term only. In
addition, any source will have some frequency spread, which
will smear out the fluctuations in the second term. If the
intensity is calculated using only the first term, a very
smooth function is obtained, from which a more meaningful
comparison of various source and receiver depths and various
ranges can be made.

This intensity gives rise to what is called the
incoherent propagation loss:

$$IPL = -10 \log_{10}\{(\pi\rho(z))^2 \sum_{n=1}^{N} |u_n(z_0)u_n(z)H_0^{(1)}(k_n r)|^2\} \qquad (19)$$

The incoherent propagation loss should be used cautiously
for deep water since the regions of high and low intensity
are averaged out. In shallow water it is a useful quantity
and for broadband sources more meaningful than the coherent
propagation loss PL.

Since the acoustic energy is trapped by the
waveguide, the propagation loss as a function of range tends
to behave as cylindrical spreading. It is sometimes convenient
to remove this geometrical term by subtracting 10 log (r)
from the propagation loss. Another geometrical factor which
can be subtracted is the water depth correction, 10 log(h).
This corresponds to the spreading of the acoustic energy

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throughout the water column, and is closely related to the spherical spreading of the acoustic energy near the point source. The remainder term contains the losses due to the depth dependence of the propagation loss and losses due to absorption by the bottom. If there is no attenuation in the bottom layer, the remainder term will be constant with range. In the printout for the program (Appendix C) the incoherent propagation loss with cylindrical spreading and water depth correction removed appears under the heading INCOH-GEOM.

The excitation of a given mode n by a source at depth z_0 will be defined to be

$$E_n = u_n(z_0) \tag{20}$$

This is the coefficient of the n-th term in the sum for the pressure, see Eq.(14).

2.2 Method of Solution

To obtain the eigenvalues k_n of the mode equation (1), the transcendental equation (7) must be solved. To facilitate the solution two dimensionless quantities are defined:

$$y_n = h[\omega^2/c_1^2 - k_n^2]^{1/2} \tag{21}$$

and

$$b = (h\omega/c_1)\ [1 - c_1^2/c_2^2]^{1/2} \tag{22}$$

Then equation (7) can be written as,

$$\tan y_n = -(\rho_2/\rho_1)\, y_n\, [b^2 - y_n^2]^{-1/2} \tag{23}$$

or equivalently,

$$y_n = n\pi - \tan^{-1} \frac{(\rho_2/\rho_1) y_n}{[b^2 - y_n^2]^{1/2}} \tag{24}$$

A Newton-Raphson iteration method is used to solve Eq. (24) for y_n. To first get a feeling for the results, a graphical solution to Eq.(23) can be obtained. This is illustrated in Figure 2 for the case of three modes. Notice that the roots y_n must occur on the negative branch of the tangent function or

$$(n - \tfrac{1}{2})\, \pi < y_n < n\pi \tag{25}$$

for $n = 1, 2, \ldots, N$, where

$$N = [[\ \tfrac{1}{2} + \tfrac{b}{\pi}\ ]] \tag{26}$$

and the notation $[[x]]$ means the greatest integer less than or equal to x. In terms of the environmental input parameters Eq.(26) becomes

$$N = [[\ \tfrac{1}{2} + (2hf/c_1)\ (1 - c_1^2/c_2^2)^{1/2}\ ]] \tag{27}$$

Also, since the right hand side of Eq.(22) is monotonically decreasing, it follows that successive values of y_n differ by less than π. So y_n is constrained by

$$(n - \tfrac{1}{2})\, \pi < y_n < \min \{n\pi,\ b,\ y_{n-1} + \pi\} \tag{28}$$

For the Newton-Raphson method we use the function

F(y) defined by

$$F(y) = y + \tan^{-1} \left[\frac{\rho_2}{\rho_1} \frac{1}{(b^2-y^2)^{1/2}} \right] \tag{29}$$

The derivative of F is then given by:

$$F'(y) = 1 + \frac{(\rho_2/\rho_1)}{(b^2-y^2)^{1/2}\{1+[(\rho_2/\rho_1)^2-1](y/b)^2\}} \tag{30}$$

and an improved estimate of y_n is obtained from the
Newton-Raphson iteration formula:

$$y_n^{(i+1)} = y_n^{(i)} - F(y_n^{(i)})/F'(y_n^{(i)}) \tag{31}$$

In the numerical procedure we must ensure that at
each iteration the new value for y_n remains in the range

$$(n - \frac{1}{2}) \pi < y_n^{(i)} < \min \{n\pi, b\} \tag{32}$$

Once y_n has been determined, the wave number is given by

$$k_n = [\omega^2/c_1^2 - y_n^2/h^2]^{1/2} \tag{33}$$

The mode functions and propagation loss, etc., can then be
calculated from the equations of Section 2.1.

3. PROGRAM DESCRIPTION

3.1 Program Structure

The program NORM2L is written in modular form for ease of expansion and modification. An attempt has been made to remain close to standard Fortran, although for input and output some straying did occur. In particular, for input of data, free field formatting was used; and for printing of text, quotes were often used instead of the Hollerith specification. A simplified flowchart of NORM2L is given in Figure 3, and the interaction of the subroutines is depicted in the tree structure of Figure 4. Appendix A is a User's Guide for program NORM2L with details on the hardware-software environment, input/output, operational instructions, restrictions and limitations, functions and subroutines, accuracy, memory requirements, and execution times.

Below is a list of the subroutines and COMMON blocks and a one line description of their functions. A short but more detailed description of the functions and subroutines appears in Appendix A.6. The COMMON blocks are discussed further in Section 3.2.

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List of Modules and COMMON Blocks

(a) Modules included in NORM2L

NORM2L   Main program. Driver program for I/O
NMODES   Calculates modes by calling MODE
MODE   Calculates eigenvalue for a single mode.
ATTENU   Calculates attenuation for the modes.
MODFUN   Prints modal functions at 12 depths
WFUNCT   Calculates mode functions at specified depths
UN   Evaluates a mode function at a specified
                depth
PLOSS   Propagation loss I/O routine
EXCITE   Calculates mode excitations for a specified
                source depth
PROPLO   Calculates propagation loss in dB//unit length
VPHI   Calculates velocity potential at the specified
                receiver depth
HANKO1   Evaluates the asymptotic Hankel function

(b) DEC-20 subroutines (not included)

DATE   Returns current date
TIME   Returns current time

(c) COMMON blocks

/CNTLIO/   I/O control. Contains logical unit numbers.
/ENVIRN/   Environmental and derived quantities
/EIGENF/   Eigenvalues and normalization constants
/CNTLNM/   Control parameters for calculating modes

COMMON block usage:

COMMON Block   Modules that use the Block

/CNTLIO/   NORM2L,NMODES,MODES,PLOSS
/ENVIRN/   NORM2L,ATTENU,MODFUN,WFUNCT,PLOSS,VPHI
/EIGENF/   NORM2L,ATTENU,WFUNCT,PLOSS,VPHI
/CNTLNM/   NORM2L,NMODES,MODES

3.2 Important variables

3.2.1 Variables in COMMON blocks

COMMON /ENVIRON/

Quantity	Variable name	Default value	Description
h	H	100.	thickness of first layer
c_1	C1	1500.	sound speed in first layer
ρ_1	RHO1	1.0	density of first layer
c_2	C2	1800.	sound speed in bottom layer
ρ_2/ρ_1	RHORAT	2.0	ratio of densities
e	ATTN	$a=0.2$	attenuation in bottom layer at given frequency
$\omega=2\pi f$	OMEGA	$f=100.$	angular frequency
$k_1=\omega/c_1$	AK1		wave number in first layer
$k_2=\omega/c_2$	AK2		wave number in bottom layer

COMMON /CNTLNM/

Variable name	Default value	Description
MAX	20	maximum number of iterations allowed in the Newton-Raphson procedure
TOL	1.0E-8	the tolerance desired in computing y_n
*NMIN	1	first mode to be calculated
NMAX	50	maximum number of modes that can be calculated
*VMIN	c_1	minimum phase velocity of modes
*VMAX	c_2	maximum phase velocity of modes

* Treated as constants in this version. "Reserved for future expansion".

COMMON /EIGENF/

Quantity	Variable name	Description
N	NM	The number of modes actually calculated
k_n	AKM(50)	Array containing wave numbers of the modes
A_n	ANM(50)	Array containing normalizations of the mode functions
δ_n	ATN(50)	Array containing imaginary part of the wave numbers

COMMON /CNTLIO/

File type	Logical variable	Current implementation Logical file	Device
Input	II	5	Terminal
Output	IT	5	Terminal
Output	IO	3	Line printer

There are three logical files for input/output although in the current version two of these are identical. Variable names are used rather than numbers to allow for compatibility with other computers and other input/output devices. For example, on DEC's PDP-1134 computer the logical file numbers are 7, 7, and 6 respectively. Only one data statement needs to be changed. It may even be possible to run the program in batch mode by letting II correspond to the card reader, IT to some arbitrary file, and IO to the line printer.

3.2.2 Other important variables

Quantity	Variable name	Default value	Description
c_2/c_1	CRAT	1.2	ratio of sound speeds
ρ_2	RHO2	$\rho_2/\rho_1=2.$	density in bottom layer
a	ALPRAT	0.2	attenuation coefficient in bottom layer (dB/length // 1kHz)
f	FREQ	100.	frequency
z_0	ZS	$h/2$	source depth
z	ZR	$h/2$	receiver depth
r	R		range of receiver
	RMIN	10000.	minimum range for propagation loss (PL) calculations
	DELR	10000.	range increment for PL calculations
	NRNG	10	number of range increments for PL calculations
v_n	VPHASE		phase velocity for mode n
PL	PL		propagation loss in dB (relative to unit distance)
IPL	IPL		incoherent PL in dB
$r.\text{IPL}$	RCOHI		IPL with cylindrical spreading removed
π	PI		Greek pi (3.14159265)
$h\,\omega/c_1$	HOMC		useful dimensionless quantity
b, b'	B, BSQ		useful quantity for determining eigenvalues (see Eq.22)

3.3   Testing and Verification

To verify that the computer program was giving
accurate answers, its output was compared with two other
sources: a HP-67 programmable hand calculator program and a
normal mode computer program by Bartberger and Ackler [9]
(to be called BANORM in this report). The HP-67 program
computes only the eigenvalues using an algorithm similar to
the one described in Section 2.2, but the calculator carries
about 10 significant digits, compared with about 8 for the
DEC-20 computer; thus its solution will be regarded as
exact. The program BANORM is a general normal mode program
which calculates eigenvalues, mode functions and propagation
loss for more complicated sound speed and density profiles
than the Pekeris model. It was chosen for comparison
because it was available, and because the CDC 6400 computer
on which it was implemented had a precision of 14
significant digits. The program used a numerical
integration of the differential equation, so its accuracy
was not known beforehand. However, as will be seen, its
eigenvalues seemed correct so its mode functions and
propagation loss were assumed to also be correct. In the
discussion below, the eigenvalues from the HP-67 program and
the eigenvalues, mode functions and propagation loss from
BANORM are compared with the corresponding values from
NORMAL.

Several modifications to the normal mode program

BANORM were necessary before the results could be compared.
First, the correction to the sound speed profile due to the
earth's curvature had to be removed. Removal of the
curvature caused the WKB approximation procedure to fail, so
an additional modification was made. Another correction
removed was the additional propagation loss due to volume
absorption in the water column. One difference which could
not be removed was the treatment of absorption in the bottom
layer. Program BANORM treats the absorption "exactly" by
using an imaginary value for the wave number, whereas in
NORM2L the absorption is treated as a perturbation. The two
programs should give identical results for the case where
there is no attenuation in the bottom. To facilitate
comparison of results BANORM was modified to accept input
and print output in metric units, although the internal
calculations were performed, as before, in yards. The values
in the tables have been modified using the exact conversion
factors as

$$k_n \rightarrow k_n/(0.9144)$$

$$u_n(z) \rightarrow (-1)^{n+1}(0.9144)^{-1/2}u_n(z)$$

$$PL \rightarrow PL + \log_{10}(0.9144)$$

Table I shows the results from the HP-67 two layer
program, with the default values for the model. These values
should have about 9 significant figures accuracy and were
used to check output from the computer program. A comparison

with the results shown in Appendix B shows excellent agreement, with eight digits accuracy in k_n.

Table II compares the eigenvalues k_n calculated by NORM2L and BANORM for the cases of no attenuation and attenuation equal to 0.2 dB/m//1 kHz. The results of the HP-67 calculation are repeated for easy comparison. For no attenuation all three methods agree to about eight significant digits, and the HP and CDC results agree to the full nine. Since the HP program did not calculate the mode functions or propagation loss, BANORM was used as the reference for judging the accuracy of NORM2L. When the bottom absorption is non-zero, the exact solution shows a shift in the real part of the eigenvalue as well as the introduction of an imaginary part. The shift in the real part is less than the imaginary part, however. In the example the real parts of k_n agree to about six figures accuracy for the exact and perturbation calculations. The imaginary parts agree to about three figures accuracy, or about the seventh decimal place. Notice that the imaginary part, though small, increases with increasing mode number - a factor of about 50 in the seven modes.

The mode functions are compared at two depths in Table III. The answers agree to about six decimal places. Presumably, BANORM is the more accurate because of the greater accuracy in the wave numbers k_n. As is quite common in numerical approximations, the eigenfunctions are less

accurate than the eigenvalues. When attenuation is considered, the mode functions agree to only four digits.

Table IV compares the propagation loss calculated by NORM2L and BANORM for source and receiver at mid-depth, with bottom absorption both present and absent. The values for the average or incoherent propagation loss (IPL) are considerably more in agreement than those for the coherent propagation loss (PL) - five digits compared with four for no absorption, and four digits compared with three digits for attenuation coefficient a=0.2. For the IPL there seems to be a consistent bias with BANORM giving values low by 0.0002 dB for a=0 and by 0.002 for a=0.2. At present no explanation can be given for this bias.

In summary, for no bottom absorption the wave numbers as calculated by NORM2L are probably accurate to eight significant digits, the mode functions accurate to about six digits, and the propagation loss to five digits. When absorption is included as a perturbation, the accuracy of the results is degraded, but the propagation loss remains accurate to better than 0.1 dB.

4. CONCLUSIONS AND RECOMMENDATIONS

NORM2L was written to provide an interactive computer program for calculating the discrete normal modes and acoustic propagation loss for the Pekeris model of the ocean. Although the model is simple, it contains all the essential features of normal mode theory, and for many shallow water environments is a reasonable approximation to the actual physical situation. The simple model has the advantage that the extensive calculations of a general normal mode program are not necessary and an interactive program can be used. The interactive nature of the program makes it useful for sensitivity studies since the user can see the results immediately, change the input parameters and look at a new set of results. The user can then quickly get a feeling for the importance of the physical parameters.

In practice the program has proved to be very useful as a learning tool for normal mode calculations and for sensitivity studies. In view of the utility of the program it is recommended that certain extensions and improvements be made to the program. Certain improvements could be made to the present program to provide more options and improve output to include graphing of mode functions and propagation loss. These are listed in Appendix E. Some extensions could be made to the model to improve its usefulness for sensitivity studies by allowing for a more realistic environment. For example, propagation loss will

be affected by the presence of shear waves in the bottom or scattering from a rough surface or bottom. These could be included as perturbation effects to be added to the present program, or treated more rigorously in an extended program. If the acoustic field is needed near the source, the continuous modes are important and should be calculated. Range dependence could be included in the model to allow for a sloping bottom or changing bottom properties. This could be incorporated in an approximate manner in the adiabatic approximation (no mode coupling), or more exactly by including mode coupling effects.

Table I. Eigenvalues obtained using the Newton-Raphson
iteration procedure

mode	iterations	y_n	k_n	v_n
1	10	2.894719966	0.417877606	1503.594646
2	9	5.805255120	0.414836757	1514.616340
3	9	8.740763033	0.409657833	1533.764231
4	8	11.70219503	0.402200817	1562.201031
5	7	14.68458504	0.392295718	1601.645144
6	7	17.67884983	0.379743941	1654.584742
7	8	20.66772252	0.364340718	1724.535581

The results shown are for the HP-67 program with the default
parameters $h=100$, $c_1=1500$, $\rho_1=1$, $c_2/c_1=1.2$, $\rho_2/\rho_1=2.0$, and
$f=100$. The results were obtained by a Newton-Raphson
procedure which iterated until successive approximations to
y_n differed by less than 1.0E-9. The initial approximation
was $y_n=(n-1/2)\pi$.

Table II. Comparison of Wave Numbers

Mode	Exact k_n	NORM2L k_n	δ_n	Bartberger-Ackler $k_n(a=0)$	$k_n(a=.2)$	$\delta_n(a=.2)$
1	.417877606	.41787760	.23081(-5)	.417877606	.417877551	.23066(-5)
2	.414836757	.41483675	.85045(-4)	.414836757	.414836565	.84998(-4)
3	.409657833	.40965783	.17149(-4)	.409657833	.409657434	.17141(-4)
4	.402200817	.40220081	.27377(-4)	.402200817	.402200241	.27364(-4)
5	.392295718	.39229571	.39711(-4)	.392295718	.392294853	.39688(-4)
6	.379743941	.37974394	.57252(-4)	.379743941	.379742482	.57202(-4)
7	.364340718	.36434072	.94016(-4)	.364340718	.364336855	.93761(-4)

The default parameters given with Table I have been used for the calculations. The exact values are from the HP-67 program.

Table III. Comparison of Mode Functions

At depth 50

mode	NORM2L	BANORM(a=0)	BANORM(a=0.2)
1	.1348502	.1348504	.1348530
2	.3228419(-1)	.3228438(-1)	.3227565(-1)
3	-.1290802	-.1290816	-.1290872
4	-.5761266(-1)	-.5761346(-1)	-.5760186(-1)
5	.1203002	.1203022	.1203128
6	.7630017(-1)	.7630139(-1)	.7628658(-1)
7	-.1085481	-.1085494	-.1085981

At depth 25

mode	NORM2L	BANORM(a=0)	BANORM(a=0.2)
1	.8997515(-1)	.8997537(-1)	.8997854(-1)
2	.1354220	.1354228	.1354246
3	.1119586	.1119598	.1119571
4	.2949192(-1)	.2949233(-1)	.2948582
5	-.6969622(-1)	-.6969738(-1)	-.6970512
6	-.1322287	-.1322308	-.1322377
7	-.1236168	-.1236184	-.1236284

The default parameters given with Table I were used for the
calculations.

Table IV.   Comparison of Propagation Loss.

a=0

| | NORM2L | | | BANORM | |
r	PL	IPL		PL	IPL
10000	57.4530	58.0333		57.4524	58.0331
20000	62.1264	61.0436		62.1259	61.0434
30000	62.9224	62.8047		62.9221	62.8043
40000	64.3649	64.0539		64.3650	64.0537
50000	81.6223	65.0230		81.6213	65.0228
60000	66.5136	65.8148		66.5127	65.8146
70000	73.5068	66.4842		73.5066	66.4841
80000	64.2857	67.0642		64.2868	67.0640
90000	72.6916	67.5757		72.6934	67.5755
100000	64.7800	68.0333		64.7807	68.0331

a=0.2

| | NORM2L | | | BANORM | |
r	PL	IPL		PL	IPL
10000	58.3779	60.0250		58.4343	60.0225
20000	64.9796	64.2338		65.1382	64.2318
30000	65.4292	66.8262		65.4717	66.8244
40000	70.6964	68.7062		70.6569	68.7044
50000	76.3537	70.1800		76.4301	70.1782
60000	75.9126	71.3920		76.0060	71.3903
70000	74.7604	72.4229		74.7096	72.4211
80000	74.8863	73.3220		74.7803	73.3202
90000	74.8971	74.1222		74.8337	74.1203
100000	74.4043	74.8458		74.3660	74.8439

The standard environmental model is used; source and
receiver are at mid-depth. PL is the coherent propagation
loss, IPL is the incoherent propagation loss, and a is the
attenuation coefficient.
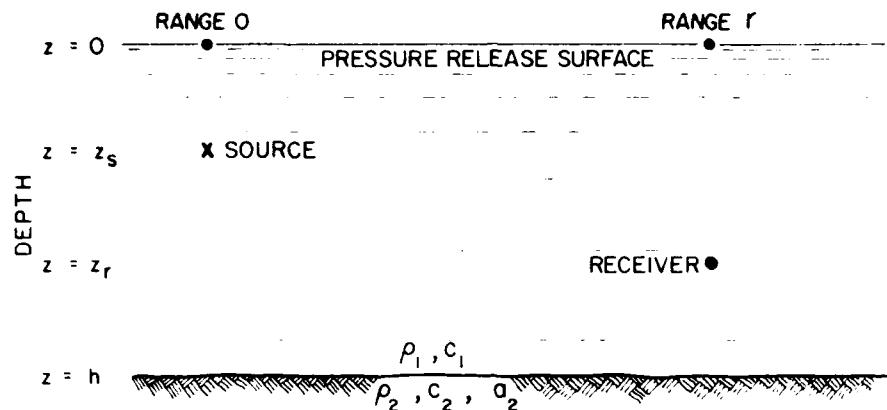
(a) The layer structure: densities ρ_1 and ρ_2, sound speeds c_1 and c_2, and bottom absorption a_2 are all constants, and $c_1 < c_2$. The source and receiver are at depths z_s and z_r respectively and separated by horizontal distance r.
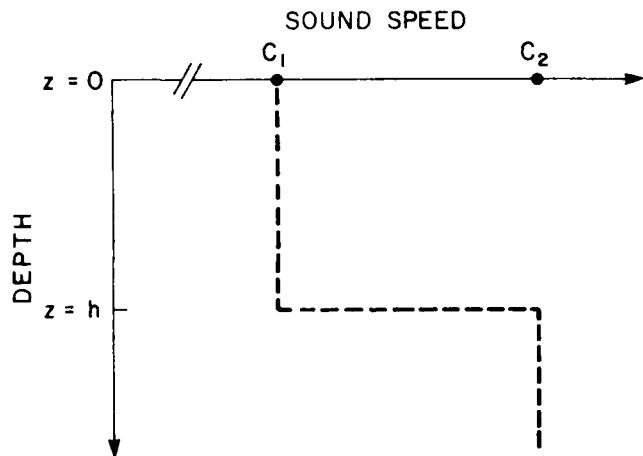
(b) The sound speed profile for the Pekeris model.

Figure 1.   The Pekeris model

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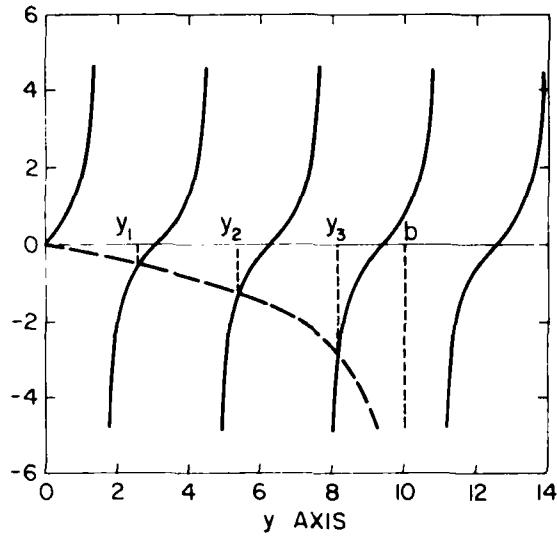
Figure 2.   Graphical Solution for the Mode Eigenvalues

Graphical solution of Eq.(23), for $\rho_2/\rho_1 = 2.0$ and $b = 10$.   The solid line is the tangent function.   The dashed line is the function: $-(\rho_1/\rho_2)y/[b^2-y^2)]^{1/2}$.   There are three modes in this example.   The solutions y_n are indicated.
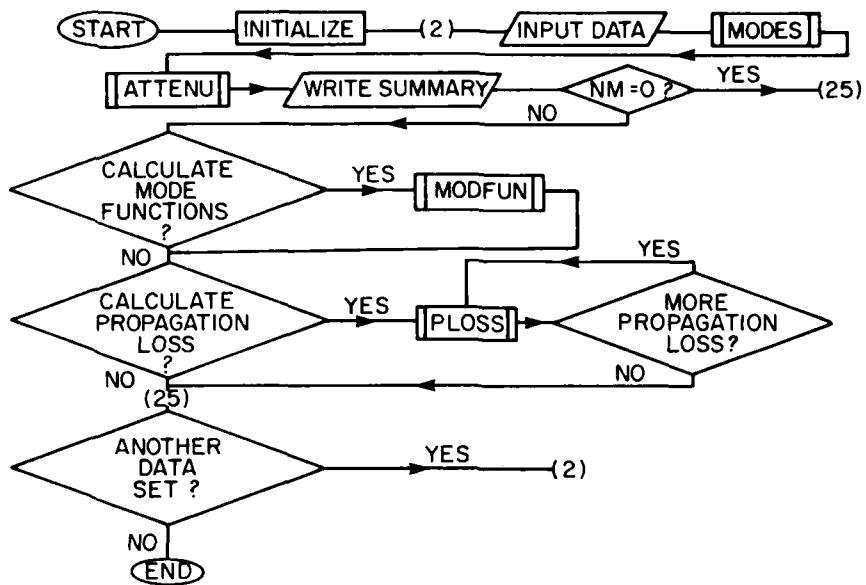
Figure 3. Simple Flowchart of NORM2L

The nodes (2) and (25) correspond to statement numbers in NORM2L.
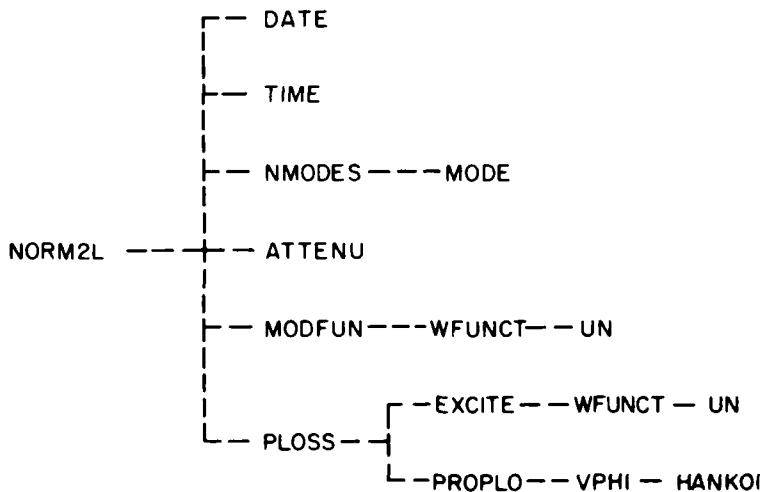
Figure 4. Tree Structure for NORM2L and its Subroutines

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5. I. Tolstoy and C.S. Clay, **Ocean Acoustics**, McGraw Hill, 1966

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9. C.L. Bartberger and L.L. Ackler, "Normal mode solutions and computer programs for underwater sound propagation: Part II Program for arbitrary velocity profiles", NADC Report No. NADC-72002-AE, April 1973

10. **Fortran Reference Manual**, DEC-20-LFRMA-A-D, Digital Equipment Corporation, Maynard MA, 1976. List directed input is described on pages 10-6 and 10-7; for the DATE and TIME subroutines see pages 15-15 and 15-18.

APPENDIX A

USER'S GUIDE FOR PROGRAM NORM2L

A.1 Identification

A.2 Hardware-Software Environment

A.3 Input/Output

A.4 Operational Instructions

A.5 Restrictions and Limitations

A.6 Functions and Subroutines

A.7 Accuracy

A.8 Memory Requirements

A.9 Execution Times

A.1 Identification

Program : NORM2L

Purpose : To calculate discrete normal modes and propagation
          loss for a two layer fluid acoustical model.

Author : Dale D. Ellis

Installation :   Defence Research Establishment Atlantic
                  P.O. Box 1012
                  Dartmouth, N.S.
                  Canada, B2Y 3Z7

Computer: DEC-20

Language: Fortran IV; with some minor DECsystem-20
          extensions for input/output.

Date written : April 1978

Last Modification : June 1978

Present version : 3.0

Security : Unclassified

A.2 Hardware-Software Environment

Hardware environment:
              - Interactive terminal
              - Line printer (optional)

Software environment:
              - Interactive operating system
              - FORTRAN IV compiler (with a few DEC-20
                extensions for input/output statements)
              - Standard Fortran library functions
              - DEC-20 subroutines for date and time.

        Note: None of the DEC-20 non-standard items are
essential to the program; they are for convenience only and
may be omitted or replaced by functions compatible with the
operating system being used. The most difficult item to
change will be the free-field format input and non-ANSI
output, but these items are becoming quite common in FORTRAN
compilers.

A.3 Input/Output

A.3.1 Input

Most input is list directed; that is no formatting is required [10].

> The exception to this is the response to a question which requires a YES or NO answer In this case no spaces are allowed. The program checks only to see if the first character is a Y, otherwise NO is assumed.

If no change is desired to a data item (other than Y/N) a null entry (not a zero) will allow its value to remain unchanged. On the DEC-20 a comma (,) is used to signify a null entry. (Note: a carriage return by itself is ignored.) The data items and their default values are:

Variable	Fortran name	Default value
depth of layer 1	H	100.
sound speed in layer 1	C1	1500.
density in layer 1	RHO1	1.0
ratio of sound speeds (layer 2 to layer 1)	CRAT	1.2
ratio of densities	RHORAT	2.0
attenuation coefficient for layer 2	ALPRAT	0.2
frequency	FREQ	100.
source depth	ZS	H/2.
receiver depth	ZR	H/2.
minimum range	RMIN	10000.
range increment	DELR	10000.
number of ranges	NRNG	10

The above values are typical units for isovelocity shallow water (layer 1) over a hard bottom (layer 2). The depths and ranges are in meters, the speeds in meters per second and the frequency in Hz. The attenuation coefficient is in dB/m at a frequency of 1 kHz. The attenuation is assumed to vary linearly with frequency. Units need not be metric but

should be consistent. In particular, the depths and ranges should be in the same units.

Some checking of the input parameters is performed but it is by no means exhaustive. All input parameters should be positive, except ALPRAT which may be zero. If $|RHO1-1.0|>0.03$ a confirmation is required. If a negative value is entered for the frequency, no calculations are performed. This enables the user who notices a previous incorrect entry to begin again. Since list directed input is used, only those values that need altering need be entered. For the others a null entry (i.e. a comma, not a zero) and carriage return will suffice.

YES/NO entries are required for control of printout and calculation of mode functions, mode excitations and propagation loss.

In the propagation loss subroutine the default values are reset each time it is called. The parameters and their default values were listed earlier. Other values may be selected and the propagation loss subroutine may be called repeatedly with different values for the source and receiver depths and range selections.

A.3.2 Output

Output consists of a summary of the input parameters for the calculation, the date and time, and results associated with the discrete normal modes. In the summary MODE is the mode number, KN is the mode wave number,

NORM is the normalization of the mode functions, IM(KN) is the imaginary part of the wave number and VPHASE is the phase speed. The output is normally displayed on the terminal. However, if YES is specified in answer to: 'DO YOU WANT A COPY OF THIS SUMMARY ON THE LINE PRINTER ?' then all further output from the calculations of the mode functions, propagation loss and mode excitations is printed on the line printer. Only the prompting questions continue to be printed on the terminal.

The mode functions, if desired, are calculated at 12 depths equally spaced by H/10, where H is the thickness of layer one. The first line of output lists the depths. The subsequent lines list the mode functions at the corresponding depths. Note that, due to the change in density, the mode functions are discontinuous at the interface between the two layers.

The mode excitations are simply the normalized mode functions evaluated at the source depth. They may optionally be printed with the propagation loss calculation.

At each range, coherent propagation loss, incoherent propagation loss, and incoherent propagation loss with cylindrical and (approximate) near-field spherical spreading removed are calculated and printed in tabular form under the headings RANGE, COH, INCOH, and INCOH-GEOM. The propagation loss subroutine can be called repeatedly with different values for the source depth, receiver depth, and range

points. Note that each time the subroutine is called the
values revert to the default values listed in the previous
section.

Sample printout from a terminal is shown in Appendix
B. The underlined characters are entered by the user. Data
entries are terminated by pressing the "return" key; this is
indicated by <ret> on the output.

Appendix C shows the line printer output
corresponding to the input shown in Appendix B. If the line
printer option is not specified, this printout will appear
at the terminal.

A.4   Operational Instructions

Initiate:

This is an interactive program; no special
instructions are necessary. For example on the DEC-20, if
the program has been SAVEd as file NORM2L.EXE on the
directory <ELLIS.NM>, then the command

@RUN <ELLIS.NM>NORM2L

will cause execution to begin. No system messages appear.
In the example of Appendix B the compiled program exists on
file NORM2L of the user's directory, and the command

@EXECUTE NORM2L

causes the relocatable files to be linked with the system
subroutines and loaded for execution. There are two lines
of system messages. If only the source file NORM2L.FOR had

been on disk, then the FORTRAN compiler would have been
called automatically and each subroutine name would have
been typed as it was compiled.

Execute:

Enter data in response to questions typed on the
terminal. See Appenxix A.3 for more details. As many data
sets as desired may be executed.

Input/output data:

Both input and output appear on the terminal.
Optionally, some output may be directed to the line
printer. See Appendices B and C for an example; responses
typed by the user are underlined.

Error procedures:

Error checking is not exhaustive although those
errors that are trapped allow the user to change user input.
Some warning messages are issued - see below under
"Messages". The DEC-20 operating system traps overflow,
square roots of negative numbers, etc., and issues warning
messages for the first two errors detected. Any system
error messages indicate that the results may be unreliable.

Interrupt:

No interrupt procedures have been coded into the
program. However, on the DEC-20 typing 'control-C' once or
twice will cause program execution to stop. The program can
be re-run with the RUN or EXECUTE command as described in

the "Initiate" paragraph above. The START or CONTINUE

commands can be used to restart the program without

reloading, but these should not be used if an error has

occurred.

Terminate:

Program execution can be terminated by typing NO

in response to the question

'DO YOU WISH TO RUN ANOTHER DATA SET ? (Y/N)'.

Messages:

The following warning messages are issued:

Message	Explanation	Response
ARE YOU SURE ABOUT RHO1? (Y/N)	$\lvert RHO1-1.\rvert > 0.03$ Usually the density is entered as the specific gravity 1.0 or 1.026. This message is to prevent a zero or ridiculous value being entered by mistake.	Answer YES if RHO1 was entered correctly; a NO will cause the previous question to be repeated.
NEGATIVE FREQUENCY. EXECUTION SUPPRESSED.	A negative number, either deliberately or by mistake, was entered for the frequency. No modes are calculated.	User is asked if he wants to run another data set.
*** WARNING *** THERE ARE [NM] MODES. ONLY [NMAX] MODES HAVE BEEN CALCULATED.	In the present version the array sizes have been dimensioned for 50 modes. Program calculates the NMAX modes and continues.	None. User must decide whether the neglected modes will affect his results.

NO TRAPPED MODES.	There are no discrete modes for the current set of environmental parameters. Further calculations are skipped.	User is asked about the usual line printer summary and whether to run another data set.
CONVERGENCE NOT ACHIEVED FOR MODE [M] AFTER [MAX] ITERATIONS, DIF= [DIF].	The Newton-Raphson procedure for mode M has failed to converge to the desired tolerance after MAX iterations The difference between the last two values of YN is DIF. Answers may be inaccurate.	Indicates TOL has been chosen too small, MAX not large enough, or a bug in the program. See section 3.3 for details on how to modify TOL or MAX.

A.5   Restrictions and Limitations

The limitations of the model are discussed in Section 2.1. This section refers to the restrictions and limitations of the program.

All input parameters must be positive; except ALPRAT which may equal zero.

The sound speed in the bottom must be greater than the sound speed in the water column; otherwise, no modes will be trapped.

The present version of the program allows for a maximum of 50 modes. This can be modified by changing a few DIMENSION statements.

A small value for the range variable may cause the Hankel function to be inaccurate. No warning is given. The error would occur at short ranges where the model is

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inaccurate anyway.

The program is written in single precision. See Appendix A.7 for more comments on accuracy.

Printout of the mode functions and propagation loss appears on the line printer or terminal, but not both.

A.6   Functions and Subroutines

A chart of the subroutine interactions is given in Section 3.1 . The program was designed to be modular for flexibility in expansion and modification; thus some duplication and inefficiency results.

The program has the following functions and subroutines. For any undefined variables see Section 3.2.

1. subroutine NMODES (H,HOMC,CRAT,RHO1,RHORAT,NM,AKM,ANM)

    - given the input environmental parameters H, HOMC, CRAT, RHO1, and RHORAT, calculates the number of trapped modes (NM), the wave numbers of the modes (array AKM), and the normalization factors for the wave functions of each mode.

2. subroutine MODE (M,YO,BSQ,RHORAT,YN,IER)

    - uses a Newton-Raphson iteration procedure to calculate YN, which gives the M-th normal mode eigenvalue [see Eqs. 29-31]. YO is an initial guess, BSQ and RHORAT some needed input variables, and IER an output error parameter in case the method fails to converge to the required tolerance.

3. subroutine ATTENU

    - calculates the imaginary part of the wave number for the modes due to the bottom attenuation using a perturbation formula. Parameters are passed using labelled common.

4. subroutine MODFUN (NM,IO)

    - calculates and prints (on logical unit number IO)
the NM eigenfunctions evaluated at the 12 depths equally
spaced at intervals of one tenth the thickness of layer one.
This subroutine should be generalized in future versions.

5. subroutine WFUNCT (N,NPT,DZ,ZVALS,WVALS,ICH)

    - evaluates and stores in array WVALS the mode
function for the N-th mode at the NPT depths stored in array
ZVALS. If ICH is non-zero the array ZVALS is calculated:
ZVALS(I)=(I-1)*DZ . The mode parameters are obtained via
labelled common.

6. function UN (Z,N,H,C1,C2,RHORAT,OMEGA,NM,AKM, ANM)

    - evaluates the N-th mode function at depth Z. The
other parameters are required for calculation of the mode
function.

7. subroutine PLOSS (IO)

    - input/output subroutine for propagation loss
calculations. IO is the logical number of the file on which
the results are written.

8. subroutine EXCITE (ZS,EX,NM)

    - calculates the mode excitations (array EX) for
source depth ZS for the first NM modes.

9. function PROPLO (R,ZR,ZS,COHI,RCOHI)

    - for range R, receiver depth ZR, and source depth
ZS, calculates the coherent propagation loss PROPLO, the
incoherent propagation loss COHI, and incoherent propagation
loss RCOHI with cylindrical and near-field spherical
spreading removed. All propagation losses are in dB
referenced to unit distance.

10. complex function VPHI (R,ZR,ZS,VPHI2)

    - evaluates the velocity potential VPHI and the
square of the incoherent velocity potential VPHI2, for range
R, receiver depth ZR and source depth ZS.

11. complex function HANK01 (RK)

- evaluates the asymptotic Hankel function of the zeroth order and first kind for large values of RK.

The following DEC-system routines are used [10]. They are not essential to the calculations and are used to uniquely label the results.

1. subroutine DATE (DDMMYY)

- places the current date as left-justified ASCII characters into a two-word array DDMMYY. The date is of the form dd-mmm-yy, where dd is a two-digit day (if the first digit is zero, it is converted to a blank), mmm is a three-character month and yy is a two-digit year; e,g. 31-Mar-78.

2. subroutine TIME (HHMM)

- returns the current time in HHMM as left-justified ASCII characters in the form hh:mm, where hh is the hours in 24-hour time and mm is the minutes.

A.7   Accuracy

The program and subroutines are written in single

precision, so the accuracy is machine dependent to some

extent. The DEC-20 has 36-bit words, or about 8 digits

accuracy for floating point numbers.

With the tolerance on the eigenvalue set at 1.0E-8,

the wave numbers seem to be correct to 8 significant

figures, the mode functions seem correct to 6 significant

figures, and for the non-attenuated case the propagation

loss seems correct to about 4 significant digits. More

extensive comparisons are made in section 3.3. In all cases

tried so far, convergence has occurred in fewer than 20 iterations.

Since only three terms are used in the expression for the Hankel function, propagation loss may be inaccurate at short ranges, on the order of several water depths. However, the normal mode model is inaccurate at these ranges anyway.

A.8   Memory Requirements

The .EXE file requires 13 pages or 6656 36-bit words on the DEC-20.

The present version of the program has 461 lines of code, a total of 12945 ASCII characters.

A.9   Execution times

The execution time depends more or less linearly on the number of modes calculated. The number of modes (see Eq. 26) in turn depends linearly on the frequency and on the thickness of layer one. The number of modes also increases with the sound speed ratio CRAT.

For the calculation of Appendices B and C, with seven modes and all options in effect, one and one-half to three CPU seconds execution time is used on the DEC-20. The exact execution time depends on the system load.

APPENDIX B

SAMPLE TERMINAL INPUT/OUTPUT

All user entries are underlined. The symbol <ret>
indicates that the user must press the "return" key to
terminate the data entry. The line printer output
associated with this example is given in Appendix C.

```
@EXECUTE NORM2L <ret>
LINK:   Loading
[LNKXCT NORM2L Execution]

14-Jun-79        09:30



                  NORMAL MODE TWO LAYER MODEL



WATER DEPTH ?
100 <ret>

FIRST LAYER: C1,RHO1 ?
1500, 1, <ret>

SECOND LAYER RATIOS: CRAT,RHORAT ?
1.2, 2.0 <ret>

                     C2,RHO2 :   1800.000        2.0

ATTENUATION IN DB/UNIT LENGTH  // 1 KHZ    ?
0.2 <ret>

FREQUENCY ?
100 <ret>

14-Jun-79        09:30



              NORMAL MODE TWO LAYER MODEL


WATER DEPTH       100.0
C1, RHO1         1500.00     1.0000
C2, RHO2         1800.00     2.0000
CRAT,RHORAT         1.2000   2.0000

FREQUENCY         100.00

ALPRAT            0.2000    DB/UNIT DISTANCE  // 1 KHZ
ATTN              0.00230   UNITS OF INVERSE DISTANCE

   7  MODES CALCULATED
```

MODE	KN	NORM	VPHASE	IM(KN)
1	0.41787760	0.135884	1503.5947	0.23081E-05
2	0.41483675	0.136394	1514.6163	0.85045E-05
3	0.40965783	0.137016	1533.7642	0.17149E-04
4	0.40220081	0.137577	1562.2010	0.27377E-04
5	0.39229571	0.137972	1601.6451	0.39711E-04
6	0.37974394	0.138101	1654.5847	0.57252E-04
7	0.36434072	0.137587	1724.5356	0.94016E-04

```
DO YOU WANT A COPY OF THIS SUMMARY ON THE LINE PRINTER ?
        TYPE YES OR NO.
Y <ret>


CALCULATE MODE FUNCTIONS ? (Y/N).
Y <ret>


CALCULATE PROPAGATION LOSS ? (Y/N).
Y <ret>


CHANGE DEFAULT VALUES ? (Y/N).
N <ret>


CALCULATE EXCITATIONS OF EACH MODE ? (Y/N)
Y <ret>


MORE PROPAGATION LOSS CALCULATIONS ? (Y/N).
Y <ret>


CHANGE DEFAULT VALUES ? (Y/N).
Y <ret>


SOURCE DEPTH, RECEIVER DEPTH ?
25,, <ret>


RMIN,DELR,NRNG ?
,,, <ret>


CALCULATE EXCITATIONS OF EACH MODE ? (Y/N)
Y <ret>


MORE PROPAGATION LOSS CALCULATIONS ? (Y/N).
N <ret>


RUN ANOTHER DATA SET ? (Y/N).
N <ret>


END OF EXECUTION
CPU TIME: 2.09  ELAPSED TIME: 1:13.20
EXIT
@
```

APPENDIX C

SAMPLE LINE PRINTER OUTPUT

The output shown here was generated by the terminal session illustrated in Appendix B.

NORMAL MODE TWO LAYER MODEL

WATER DEPTH	100.0	
C1, RHO1	1500.00	1.0000
C2, RHO2	1800.00	2.0000
CRAT,RHORAT	1.2000	2.0000

FREQUENCY   100.00

ALPRAT	0.2000	DB/UNIT DISTANCE // 1 KHZ
ATTN	0.00230	UNITS OF INVERSE DISTANCE

   7   MODES CALCULATED

MODE	KN	NORM	VPHASE	IM(KN)
1	0.41787760	0.135884	1503.5947	0.23081E-05
2	0.41483675	0.136394	1514.6163	0.85045E-05
3	0.40965783	0.137016	1533.7642	0.17149E-04
4	0.40220081	0.137577	1562.2010	0.27377E-04
5	0.39229571	0.137972	1601.6451	0.39711E-04
6	0.37974394	0.138101	1654.5847	0.57252E-04
7	0.36434072	0.137587	1724.5356	0.94016E-04

MODE WAVE FUNCTIONS

DEPTHS

10.00000	,	20.00000	,	30.00000	,	40.00000	,
50.00000	,	60.00000	,	70.00000	,	80.00000	,
90.00000	,	100.0000	,	110.0000	,	120.0000	,

MODES
 1
```
 0.3878757E-01,  0.7434761E-01,  0.1037211     ,  0.1244640     ,
 0.1348502     ,  0.1340154     ,  0.1220292     ,  0.9988882E-01,
 0.6943669E-01,  0.3320670E-01,  0.1669175E-02,  0.1678062E-03,
 2
 0.7480736E-01,  0.1251040     ,  0.1344102     ,  0.9967681E-01,
 0.3228419E-01, -0.4568638E-01, -0.1086878     , -0.1360775     ,
-0.1188813     , -0.6273356E-01, -0.3334304E-02, -0.3544382E-03,
 3
 0.1050843     ,  0.1348664     ,  0.6800471E-01, -0.4758839E-01,
-0.1290802     , -0.1180745     , -0.2245802E-01,  0.8925168E-01,
 0.1370046     ,  0.8658157E-01,  0.5072421E-02,  0.5943403E-03,
```

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4
0.1266855   , 0.9880192E-01, -0.4962995E-01, -0.1375083   ,
-0.5761266E-01, 0.9257621E-01, 0.1298128   , 0.8664670E-02,
-0.1230552   , -0.1046353   , -0.7094869E-02, -0.9621448E-03,
5
0.1372500   , 0.2804273E-01, -0.1315203   , -0.5491478E-01,
0.1203002   , 0.7949435E-01, -0.1040581   , -0.1007553   ,
0.8347185E-01, 0.1178102   , 0.9832619E-02, 0.1641291E-02,
6
0.1354279   , -0.5303766E-01, -0.1146567   , 0.9794070E-01,
0.7630017E-01, -0.1278222   , -0.2624115E-01, 0.1380990   ,
-0.2784262E-01, -0.1271950   , -0.1425772E-01, -0.3196392E-02,
7
0.1210085   , -0.1151735   , -0.1138861E-01, 0.1260130   ,
-0.1085481   , -0.2269905E-01, 0.1301526   , -0.1011776   ,
-0.3385370E-01, 0.1333989   , 0.2348356E-01, 0.8268099E-02,

PROPAGATION LOSS

MODE EXCITATIONS   SOURCE DEPTH = 50.00
1   0.13485018
2   0.32284186E-01
3   -0.12908018
4   -0.57612656E-01
5   0.12030022
6   0.76300173E-01
7   -0.10854806

SOURCE DEPTH   50.00   RECEIVER DEPTH   50.00

RANGE	P.L. (DB RE UNIT DISTANCE)		
	COH	INCOH	INCOH-GEOM
10000.00	58.37791	60.02504	0.2504134E-01
20000.00	64.97957	64.23382	1.223522
30000.00	65.42920	66.82623	2.055018
40000.00	70.69642	68.70615	2.685551
50000.00	76.35365	70.17996	3.190257
60000.00	75.91258	71.39203	3.610515
70000.00	74.76040	72.42286	3.971880
80000.00	74.88633	73.32203	4.291131
90000.00	74.89706	74.12217	4.579747
100000.0	74.40432	74.84582	4.845824

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PROPAGATION LOSS

MODE EXCITATIONS   SOURCE DEPTH = 25.00
1   0.89975154E-01
2   0.13542200
3   0.11195862
4   0.29491923E-01
5   -0.69696217E-01
6   -0.13222866
7   -0.12361683

SOURCE DEPTH   25.00   RECEIVER DEPTH   50.00

| RANGE | | P.L. (DB RE UNIT DISTANCE) | | | |
		COH		INCOH	INCOH-GEOM
10000.00	,	67.96269	,	62.09322	,	2.093222	,
20000.00	,	66.87508	,	66.65010	,	3.639804	,
30000.00	,	71.06421	,	69.42616	,	4.654949	,
40000.00	,	73.10997	,	71.44136	,	5.420760	,
50000.00	,	69.76201	,	73.03228	,	6.042584	,
60000.00	,	70.69862	,	74.34834	,	6.566830	,
70000.00	,	72.88694	,	75.47003	,	7.019045	,
80000.00	,	75.62021	,	76.44690	,	7.416999	,
90000.00	,	78.87690	,	77.31236	,	7.769934	,
100000.0	,	80.43400	,	78.09026	,	8.090258	,

APPENDIX D

PROGRAM LISTING

```
      PROGRAM NORM2L
C
C  TWO LAYER NORMAL MODE PROGRAM
C
C          AUTHOR: DALE D. ELLIS
C                  DEFENCE RESEARCH ESTABLISHMENT ATLANTIC
C                  DARTMOUTH, NOVA SCOTIA, CANADA
C
C          DATE WRITTEN: APRIL 1978
C          LAST MODIFICATION: JUNE 1978
C              ( SOME COSMETIC CHANGES SINCE THEN)
C          VERSION : 3.0    MOD 41
C          COMPUTER: DEC-20/40
C          OPERATING SYSTEM: TOPS-20
C
C          LANGUAGE: FORTRAN IV, WITH A FEW DEC-SYSTEM EXTENSIONS
C
C
C
      COMMON /ENVIRN/ H,C1,RHO1,C2,RHORAT,ATTN,OMEGA,AK1,AK2
      COMMON /EIGENF/ NM,AKM(50),ANM(50),ATN(50)
      COMMON /CNTLNM/ MAX,TOL,NMIN,NMAX,VMIN,VMAX
      COMMON /CNTLIO/ II,IO,IT
      DIMENSION DDMMYY(2)
      DATA H,C1,RHO1,CRAT,RHORAT,ALPRAT,FREQ/100.,1500.,1.,1.2
     1, 2.0,0.2,100. /
      DATA  II,IO,IT  /5,3,5/
      DATA  TOL   /1.E-8/, MAX/20/, NMIN/1/, NMAX/50/
      DATA NCALL/0/
C
      PI2 = 8.*ATAN(1.)
      ISAVE=IO
      DBNP=ALOG(10.)/20.
      CALL DATE (DDMMYY)
      CALL TIME (HHMM)
      WRITE (IT,500) DDMMYY,HHMM
500   FORMAT (1X,2A5,5X,A5)
C
C        INPUT  DATA
C
    2 CONTINUE
C
      WRITE (IT,400)
400   FORMAT (///,11X,28H NORMAL MODE TWO LAYER MODEL )
C
      WRITE (IT,401)
401   FORMAT (///,14H WATER DEPTH ?)
      READ (II,*)  H
C
    4 WRITE (IT,402)
402   FORMAT (24H FIRST LAYER: C1,RHO1 ? )
      READ (II,*)   C1,RHO1
```

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```
          IF(ABS(RHO1-1.) .LT.0.03) GO TO 5
          WRITE(IT,412)
412       FORMAT (' ARE YOU SURE ABOUT RHO1 ? (Y/N).')
          READ (II,302) YESNO
          IF(YESNO.NE.1HY) GO TO 4
        5 WRITE (IT,403)
403       FORMAT (36H SECOND LAYER RATIOS: CRAT,RHORAT ? )
          READ (II,*)  CRAT,RHORAT
          IF (CRAT.GT.1.) GO TO 6
          WRITE (IT,417)
417       FORMAT (' CRAT=C2/C1  MUST BE GREATER THAN 1.0. ENTER NEW VALUE.')
          GO TO 5
        6 RHO2 = RHO1*RHORAT
          C2  = CRAT * C1
          WRITE (IT, 404) C2,RHO2
404       FORMAT (22X,10HC2,RHO2 : ,F10.3,F10.1,/)
C
          WRITE (IT,406)
406       FORMAT (' ATTENUATION IN DB/UNIT LENGTH  // 1 KHZ   ?')
          READ (II,*) ALPRAT
C
          WRITE (IT,405)
405       FORMAT (12H FREQUENCY ?)
          READ (II,*) FREQ
          OMEGA =PI2*FREQ
          IF (FREQ.LT.0.) WRITE (IT,413)
413       FORMAT (' NEGATIVE FREQUENCY.  EXECUTION SUPPRESSED.')
          IF (FREQ.LT.0.) GO TO 25
C
          HOMC= H*OMEGA/C1
          AK1 = OMEGA/C1
          AK2 = OMEGA/C2
          ATTN=DBNP*(ALPRAT*FREQ/1000.)
C
          CALL NMODES (H,HOMC,CRAT,RHO1,RHORAT,NM,AKM,ANM)
C
          IF (NM.LE.NMAX) GO TO 8
          WRITE (IT,419) NM,NMAX
419       FORMAT(//,' ***  WARNING  ***   THERE ARE',I5,' MODES',/
         1,' ONLY',I5,' MODES HAVE BEEN CALCULATED.')
          NM=NMAX
        8 IF (NM.GT.0.)  CALL ATTENU
          IO=IT
          IC=0
        9 CONTINUE
          IF(NCALL.NE.0) WRITE (IO,499)
499       FORMAT(1H1)
          WRITE (IO,500) DDMMYY,HHMM
          WRITE (IO,400)
          WRITE (IO,501) H,C1,RHO1,C2,RHO2,CRAT,RHORAT,FREQ
         1, ALPRAT,ATTN
          WRITE (IO,507) NM
```

```
507    FORMAT (/,I5,'  MODES CALCULATED')
       IF (NM.LT.1) GO TO 13
       WRITE (IO,510)
510    FORMAT (//,6H  MODE,7X,2HKN,13X,4HNORM,10X,6HVPHASE,9X,
      1 6HIM(KN) )
C
       DO 11 I=1,NM
       VPHASE=OMEGA/AKM(I)
       WRITE (IO,301)  I,AKM(I),ANM(I),VPHASE,ATN(I)
    11 CONTINUE
301    FORMAT (I5,G17.8,G15.6,G16.8,G13.5)
501    FORMAT (12H-WATER DEPTH,F10.1,/,12H C1, RHO1   ,F11.2,
      1 F11.4,/,12H C2, RHO2   ,F11.2,F11.4,/,12H CRAT,RHORAT,
      2 F13.4,F9.4,//,12H FREQUENCY  ,F11.2 ,
      3  //,7H ALPRAT,F18.4,'   DB/UNIT DISTANCE  // 1 KHZ',
      4  /,5H ATTN,11X,F10.5,'  UNITS OF INVERSE DISTANCE')
    13 IF (IC.EQ.1)  GO TO 15
       WRITE (IT,407)
407    FORMAT (//,' DO YOU WANT A COPY OF THIS SUMMARY ON THE LINE',
      1' PRINTER ?',/,10X,'TYPE YES OR NO.')
       READ (IT,302) YESNO
302    FORMAT (A1)
       IF (YESNO.NE.1HY) GO TO 15
       IO=ISAVE
       IC=1
       NCALL=NCALL+1
       GO TO 9
    15        CONTINUE
       IF (NM.LT.1) GO TO 25
C       IO=ISAVE
       WRITE (IT,408)
408    FORMAT (' CALCULATE MODE FUNCTIONS ? (Y/N).')
       READ (IT,302)  YESNO
       IF (YESNO.NE.1HY)  GO TO 20
       CALL MODFUN (NM,IO)
    20 WRITE (IT,409)
409    FORMAT( ' CALCULATE PROPAGATION LOSS ? (Y/N).')
       READ (IT,302) YESNO
       IF (YESNO.NE.1HY ) GO TO 25
    22 CALL PLOSS (IO)
       WRITE (IT,415)
415    FORMAT (' MORE PROPAGATION LOSS CALCULATIONS ? (Y/N).')
       READ (IT,302) YESNO
       IF (YESNO.EQ.1HY) GO TO 22
    25 WRITE (IT,410)
410    FORMAT(' RUN ANOTHER DATA SET ? (Y/N).')
       READ (IT,302)  YESNO
       CALL TIME (HHMM)
       IF (YESNO.EQ.1HY) GO  TO 2
       END
```

```
      SUBROUTINE NMODES (H,HOMC,CRAT,RHO1,RHORAT,NM,AKM,ANM)
C
C  NORMAL MODE CALCULATION FOR TWO LAYERS
C
C      WRITTEN BY  DALE D ELLIS    APRIL 1978
C
      COMMON /CNTLNM/MAX,TOL,NMIN,NMAX,VMIN,VMAX
      COMMON /CNTLIO/ II,IO,IT
      LOGICAL EXTRA
      DIMENSION  AKM(NM),ANM(NM)
      DATA PI /3.14159265 /
      DATA EXTRA /.FALSE./
      BSQ = HOMC **2  * (1.-1./CRAT**2 )
      B   = SQRT (BSQ)
      NM  = B/PI + 0.5
      IF (NM.LT.1) GO TO 40
      NMX = MINO (NM,NMAX)
      YO  = (NMIN-1.25)*PI
      DO 20 M =NMIN,NMX
      YO = YO+PI
      IF(EXTRA) WRITE(IT,410) M
410   FORMAT (5HOMODE,I5)
      IM = M
      CALL MODE (IM,YO,BSQ,RHORAT,YN,IER)
      RKNSQ= (HOMC**2-YN**2)/H**2
      RKN  = SQRT (RKNSQ)
      AKM(M)=RKN
      GM1H = YN
      GM2H = SQRT ( BSQ-YN**2)
      AM2  = 2./H/RHO1/(1.-SIN(2.*GM1H)/(2.*GM1H) +(SIN (GM1H))
     1**2/(RHORAT*GM2H))
      ANM(M)=SQRT(AM2)
      IF (EXTRA) WRITE (IT,*) YN,AKM(M),ANM(M)
   20 CONTINUE
C
      RETURN
C
   40 WRITE(IT,420)
420   FORMAT (17H NO TRAPPED MODES )
      RETURN
      END
```

```fortran
      SUBROUTINE MODE (M,YO,BSQ,RHORAT,YN,IER)
C
C OBTAINS YM FOR M-TH MODE USING NEWTON-RAPHSON PROCEDURE
C
C      WRITTEN BY  DALE D ELLIS     APRIL 1978
C
      LOGICAL EXTRA
      COMMON /CNTLNM/ MAX,TOL,NMIN,NMAX,VMIN,VMAX
      COMMON /CNTLIO/ II,IO,IT
      DATA PI /3.14159265358979/
      DATA EXTRA /.FALSE./
      IER=0
      B=SQRT(BSQ)
      R21B=(RHORAT**2-1.)/BSQ
      PIM=PI*M
      YMAX=AMIN1(PIM,B)
      YMIN=PI*(M-0.5)
      YM=YO
      IF (YM.GT.YMAX.OR.YM.LT.YMIN) YM=(YMIN+YMAX)/2.
C
      DO 10 1=1,MAX
      SRBY = SQRT (BSQ-YM**2)
      FY = YM-PIM + ATAN (RHORAT*YM/SRBY)
      DF = 1. + RHORAT/SRBY/(1. + R21B*Y**2)
      YN = YM-FY/DF
      DIF= YN-YM
      IF (EXTRA) WRITE(IT,*) YM,FY,DF,YN,DIF
      IF (ABS(DIF/YN).LT. TOL) GO TO 15
      YM=YN
      IF (YM.LT.YMIN) YM=YMIN
      IF (YM.GT.YMAX) YM=YMAX
   10 CONTINUE
C
      WRITE (IT,410) M,MAX,DIF
  410 FORMAT (34H CONVERGENCE NOT ACHIEVED FOR MODE ,I3,/,10X,
     1 5HAFTER,I3,19H ITERATIONS DIF =, E12.4)
      IER=1
      RETURN
C
   15 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ATTENU
C
C  BOTTOM ATTENUATION USING INGENTO'S PERTURBATION FORMULA
C
C      WRITTEN BY  DALE D ELLIS     APRIL 1978
C
      COMMON /ENVIRN/ H,C1,RHO1,C2,RHORAT,ATTN,OMEGA,AK1,AK2
      COMMON /EIGENF/ NM,AKM(50),ANM(50),ATN(50)
C
      IF (ATTN.EQ.0.) GO TO 20
C
      CONST=0.5*ATTN*OMEGA*RHO1/C2/RHORAT
C
      DO 10 I=1,NM
      ATN(I) = CONST*(ANM(I)*SIN(H*SQRT(AK1**2-AKM(I)**2)))**2
     1 / (AKM(I)*SQRT(AKM(I)**2-AK2**2) )
   10 CONTINUE
      RETURN
C
C          NO ATTENUATION
   20 DO 21 I=1,NM
   21 ATN(I)=0.
      RETURN
      END
```

```fortran
      SUBROUTINE MODFUN (NM,IO)
C
C  INPUT/OUTPUT ROUTINE FOR MODE FUNCTION CALCULATIONS
C
C        WRITTEN BY  DALE D ELLIS     APRIL 1978
C
      COMMON /ENVIRN/ H,C1,RHO1,C2,RHORAT,ATTN,OMEGA,AK1,AK2
      DIMENSION ZVALS(100),WVALS(100)
      WRITE (IO,430)
430   FORMAT(////,10X,'MODE WAVE FUNCTIONS',/)
      NPT=12
      IN=1
      DZ=H/10.
      CALL WFUNCT (1,NPT,DZ,ZVALS,WVALS,0)
      WRITE (IO,431)
431   FORMAT (' DEPTHS')
      WRITE (IO,*) (ZVALS(I),I=1,NPT)
      WRITE (IO,432)
432   FORMAT (/,' MODES')
      WRITE (IO,202) IN
202   FORMAT (I3)
      WRITE (IO,*) (WVALS(I),I=1,NPT)
      IF (NM.LT.2) RETURN
      DO 10 IN=2,NM
      INN=IN
      CALL WFUNCT (INN,NPT,DZ,ZVALS,WVALS,0)
      WRITE (IO,202)  INN
      WRITE (IO,*) (WVALS(I),I=1,NPT)
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE WFUNCT (N,NPT,DZ,ZVALS,WVALS,ICH)
C
C EVALUATES MODE FUNCTIONS AT A NUMBER OF DEPTHS
C
C      WRITTEN BY  DALE D ELLIS    APRIL 1978
C
      COMMON /ENVIRN/ H,C1,RHO1,C2,RHORAT,ATTN,OMEGA,AK1,AK2
      COMMON /EIGENF/ NM,AKM(50),ANM(50),ATN(50)
      DIMENSION ZVALS(NPT),WVALS(NPT)
      IF (ICH.NE.0)  GO TO 10
      DO 4 I=1,NPT
    4 ZVALS (I)=DZ*I
   10 CONTINUE
      DO 14 I=1,NPT
      WVALS(I) = UN (ZVALS(I),N,H,C1,C2,RHORAT,OMEGA,NM,AKM,ANM )
   14 CONTINUE
      RETURN
      END




      FUNCTION UN (Z,N,H,C1,C2,RHORAT,OMEGA,NM,AKM,ANM )
C
C N-TH NORMAL MODE EVALUATED AT DEPTH Z
C
C      WRITTEN BY  DALE D ELLIS    APRIL 1978
C
      DIMENSION AKM(NM),ANM(NM)
      IF (N.GT.NM) GO TO 13
      GM1=SQRT ( (OMEGA/C1)**2 - AKM(N)**2 )
      IF (Z.GT.H) GO TO 4
      UN = ANM(N)*SIN (GM1*Z)
      RETURN
    4 UN=1./RHORAT*ANM(N)*EXP(-(Z-H)*SQRT(AKM(N)**2-(OMEGA/C2)**2))
    1 * SIN (GM1*H)
      RETURN
   13 UN=0.
      RETURN
      END
```

```
      SUBROUTINE PLOSS (IO)
C
C  INPUT/OUTPUT ROUTINE FOR PROPAGATION LOSS CALCULATIONS
C
C       WRITTEN BY  DALE D ELLIS      APRIL 1978
C
      COMMON /ENVIRN/ H,C1,RHO1,C2,RHORAT,ATTN,OMEGA,AK1,AK2
      COMMON /EIGENF/ NM,AKM(50),ANM(50),ATN(50)
      COMMON /CNTL10/ II,ION,IT
      DIMENSION EX(50)
      WRITE (IO,440)
440   FORMAT (///,' PROPAGATION LOSS'/)
      ZS=H/2.
      ZR=H/2.
      RMIN=10000.
      DELR=10000.
      NRNG=10
      WDEPTH= 10.*ALOG10(H)
C
      WRITE (IT,434)
434   FORMAT (' CHANGE DEFAULT VALUES ? (Y/N).')
      READ(IT,435) YESNO
435   FORMAT (A1)
      IF(YESNO.NE.1HY) GO TO 6
      WRITE (IT,436)
436   FORMAT (' SOURCE DEPTH, RECEIVER DEPTH ?')
      READ (IT,*) ZS,ZR
      WRITE (IT,437)
437   FORMAT (' RMIN,DELR,NRNG ?')
      READ (IT,*) RMIN,DELR,NRNG
C
    6 CONTINUE
      WRITE (IT,438)
438   FORMAT (' CALCULATE EXCITATIONS OF EACH MODE ? (Y/N)')
      READ (IT,435) YESNO
      IF (YESNO.NE.1HY) GO TO 8
      CALL EXCITE (ZS,EX,NM)
      WRITE (IO,439) ZS
439   FORMAT (20X,'MODE EXCITATIONS',10X,'SOURCE DEPTH =',G11.4)
      WRITE (IO,444)  (I,EX(I),I=1,NM)
444   FORMAT (15X,I5,G17.8)
    8 CONTINUE
C
      WRITE (IO,441)  ZS,ZR
441   FORMAT (/,13H SOURCE DEPTH,F10.2,10X,14HRECEIVER DEPTH,
     1 F10.2,//,5X,5HRANGE,13X,'P.L. (DB RE UNIT DISTANCE)')
      WRITE (IO,442)
442   FORMAT (2iX,'COH',12X,'INCOH',9X,'INCOH-GEOM')
      DO 10 IR=1,NRNG
      R=RMIN+(IR-1)*DELR
      PL=PROPLO (R,ZR,ZS,COHI,RCOHI)
      GCOHI=RCOHI-WDEPTH
```

```
      WRITE (10,*) R,PL,COHI,GCOHI
   10 CONTINUE
      RETURN
      END




      SUBROUTINE EXCITE (ZS,EX,NMD)
C
C  CALCULATES MODE EXCITATIONS (I.E. MODE FUNCTION AT SOURCE DEPTH)
C
C        WRITTEN BY  DALE D ELLIS      APRIL 1978
C
      DIMENSION EX(NMD)
      DO 10 I =1,NMD
      II=I
      CALL WFUNCT (II,1,DZ,ZS,EX(II),1)
   10 CONTINUE
      RETURN
      END




      FUNCTION PROPLO (R,ZR,ZS,COHI,RCOHI)
C
C  PROPAGATION LOSS IN DB
C
C        WRITTEN BY  DALE D ELLIS      APRIL 1978
C        REVISED JUNE 1979 - QUANTITIES MADE POSITIVE
C
      COMPLEX VPHI,V
      V = VPHI (R,ZR,ZS,VCOHI)
      VMOD = V*CONJG (V)
      PROPLO= -10.*ALOG10 (VMOD)
      COHI  = -10.*ALOG10(VCOHI)
      RCOHI = -10.*ALOG10(R*VCOHI)
      RETURN
      END
```

```
      COMPLEX FUNCTION VPHI (R,ZR,ZS,COHI)
C
C  VELOCITY POTENTIAL FOR PROPAGATION LOSS CALCULATIONS
C   ALLOWS RECEIVER DEPTH ZR TO BE GREATER THAN WATER DEPTH H
C
C       WRITTEN BY  DALE D ELLIS      APRIL 1978
C
      COMPLEX HANKO1,HK,TERM
      DOUBLE PRECISION SUMR,SUMI
      COMMON /ENVIRN/ H,C1,RHO1,C2,RHORAT,ATTN,OMEGA,AK1,AK2
      COMMON /EIGENF/ NM,AKM(50),ANM(50),ATN(50)
      DATA PI/3.14159265/
C
      SUMR=0.D0
      SUMI=0.D0
      COHI=0.
      DO 10 I=1,NM
      HK=HANKO1(AKM(I)*R)
      GM1=SQRT (AK1**2-AKM(I)**2)
      UNS =SIN (ZS*GM1) *ANM(I)
      ALOSS=EXP(-ATN(I)*R)
      IF(ZR.GT.H) GO TO 6
      UNR =SIN (ZR*GM1) *ANM(I)
      GO TO 8
    6 GM2= SQRT (AKM(I)**2-AK2**2)
      UNR =1./ RHORAT*ANM(I)*EXP(-(ZR-H)*GM2) * SIN (GM1*H)
    8 SUMR = SUMR + UNS*UNR*ALOSS*REAL(HK)
      SUMI = SUMI + UNS*UNR*ALOSS*AIMAG(HK)
      TERM=(UNS*UNR*ALOSS)*HK
      COHI=TERM*CONJG(TERM) + COHI
   10 CONTINUE
C
      VPHI = CMPLX(0.,PI)*CMPLX(SNGL(SUMR),SNGL(SUMI))
      COHI=PI**2 * COHI
      RETURN
      END
```

```
      COMPLEX FUNCTION HANK01(Z)
C
C  ASYMPTOTIC HANKEL FUNCTION OF FIRST KIND AND ZEROTH ORDER
C
C      WRITTEN BY  DALE D ELLIS     APRIL 1978
C
      DATA PI /3.14159265/
      CHI = Z - PI/4.
      P = 1. - (9./128.)/Z**2
      Q = -1./(8*Z)
C
      HANK01 = SQRT(2./(PI*Z))*CMPLX(P,Q)*CMPLX(COS(CHI),SIN(CHI))
      RETURN
      END
```

APPENDIX E

FUTURE MODIFICATIONS AND EXTENSIONS

Some suggested improvements and extensions to the present program are listed below.

1. Make optional instructions available to the user at the beginning of program execution.

2. Type the current value of the data item to be entered. For example, the first question might read:
       'WATER DEPTH   100.   ?'

3. Add a subroutine CHOICE to allow the user to modify some of the control parameters in the COMMON block /CNTLNM/. For the option of starting the calculations at a mode number greater than 1, some subroutines would need modification.

*4. Improve the Hankel function to maintain accuracy near the source. Type a warning message when the function is not sufficiently accurate.

5. Generalize MODFUN to handle an arbitrary number of specified depths. The subroutines called by MODFUN are sufficiently general to handle this possibility.

6. Add the possibility of the optional printout appearing on both the line printer and terminal.

*7. Calculate the group velocity, cutoff frequency, and equivalent ray angle for each mode.

*8. Optionally display the mode functions and propagation loss, either as a line printer plot or a proper graph.

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* These items have been included in a newer version of NORM2L - PEKRIS - which is still undergoing development.

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It is hoped to try some new ideas using NORM2L or a revised version of it.

1. Incorporate range dependent normal mode theory, at least in the adiabatic approximation.

2. Include the effects of scattering from a rough surface or bottom.

3. Add the effects of continuous modes. This is necessary if pressure or propagation loss is needed near the source.

4. Create a similar program in which the bottom layer will also support shear waves.

5. Already parts of the program have been modified and used in a problem involving mode enhancement [4], and for debugging a general normal mode program [3]. These uses were not foreseen at the time of writing the program.

APPENDIX F
USER COMMENTS

This space is provided for the user to add his notes or comments. Any suggestions for improvements or additions will be considered by the author for implementation in future versions of NORM2L.

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13. ABSTRACT

The interactive computer program, NORM2L, calculates the discrete normal modes and acoustic propagation loss for the Pekeris model of the ocean. The Pekeris model is a simple two-layer model in which the two layers represent the seawater and seabed. For many shallow-water environments, the model is a reasonable approximation to the actual physical situation and can be used to investigate acoustic propagation at low frequencies.

For ease of future expansion and modification, the program NORM2L is written in modular form in FORTRAN. The results of NORM2L are compared with those of other computer programs.

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KEY WORDS

COMPUTER PROGRAM

NORMAL MODES

UNDERWATER ACOUSTICS

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